REPORT No. 665

CALCULATION OF THE AERODYNAMIC CHARACTERISTICS OF TAPERED WINGS WITH PARTIAL-SPAN FLAPS

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SUMMARY

Factors derived from wing theory are presented. By means of these factors, the angle of zero lift, the lift-curve slope, the pitching moment, the aerodynamic-center position, and the induced drag of tapered wings with partial-span flaps may be calculated. The factors are given for wings of aspect ratios 6 and 10, of taper ratios from 0.25 to 1.00, and with flaps of various lengths.

An example is presented of the method of application of the factors. Fair agreement with experimental results is shown for two wings of different taper ratio having plain flaps of various spans.

INTRODUCTION

Because of the widespread use of tapered wings equipped with partial-span flaps, it is desirable to have means for computing their aerodynamic characteristics. Previous reports (references 1, 2, and 3) have presented theoretical factors for use in computing the aerodynamic characteristics of wings with linear and with arbitrary twist and for use in finding the load distribution of wings with partial-span flaps.

This report presents factors, based on airfoil theory, for use in calculating the induced drag, the angle of zero lift, the pitching moment, and the aerodynamic center of tapered wings with partial-span flaps of constant flap-chord ratio. The factors, when used with adequate section data, should apply to various types of flap and various amounts of flap deflection.

THEORETICAL RESULTS

The particular wing chord distributions for which the theoretical computations were specifically made are given in figure 1 where the wing quarter-chord line is shown as straight. Two aspect ratios (A=6 and 10) and four taper ratios (λ=1.00, 0.75, 0.50, and 0.25) were used. A list of the symbols used herein is given in appendix A. Inasmuch as the various characteristics for elliptical wings with partial-span flaps could be obtained relatively easily, they were sometimes computed in order to aid in determining the shape of the various computed curves for the tapered wings.

The span load distributions from which the aerodynamic characteristics were obtained are given in reference 2 where a slope of the section lift curve equal to 5.67 per radian was used. The computations apply only to those cases in which no aerodynamic twist is present before the flaps are deflected.

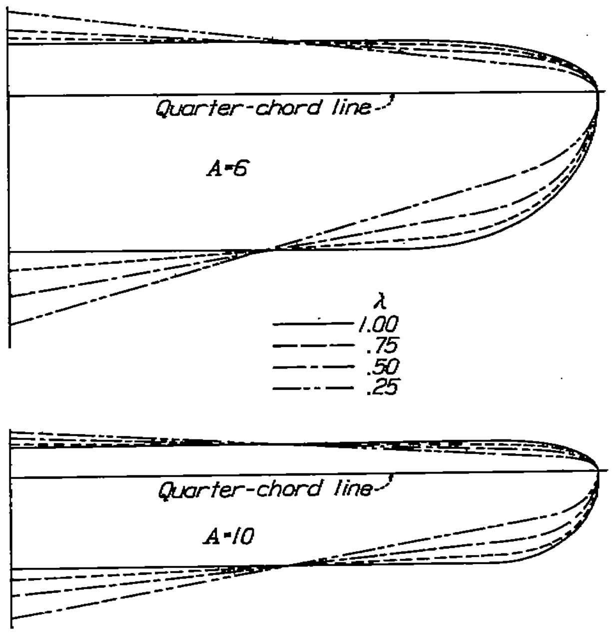

Figure 1.—Wing chord distributions.

Although the ordinary lifting-line theory is applicable only to wings without sweepback, experimental evidence indicates that small amounts of sweepback have no appreciable effect on the span loading. The computations may thus be applied to wings with moderate sweepback as long as the chord distributions are similar to those indicated in figure 1.

The computed aerodynamic characteristics are given in terms of factors such as J, H, and G. The method of calculating the factors is omitted because of its length, but the formulas for the factors are presented in appendix B. The physical significance of the factors and of the aerodynamic characteristics they represent, however, is explained in the following sections.

Angle of zero lift.—The change in the angle of zero lift of a finite wing accompanying a flap deflection depends upon several variables, such as flap span, flap deflection, flap chord, and flap type. The effect of the last three variables can be conveniently represented by the section characteristic Δc_l, the increment of section lift coefficient obtained by deflecting the flap.

By this grouping of variables, the change in the angle of zero lift (in degrees) for a wing with partial-span flaps can be expressed by the equation

$$\Delta\alpha_{s_{(L=0)}} = -J\Delta c_l \tag{1}$$

In order to obtain the angle of zero lift for the wing, this increment must be added to the initial angle of zero lift, i. e., the angle before the flap is deflected. If this initial angle is measured from the chord of the root section, as is usually the case, the angle of zero lift for the wing is given by

$$\alpha_{s_{(L=0)}} = \alpha_{l_{0_s}} - J\Delta c_l \tag{2}$$

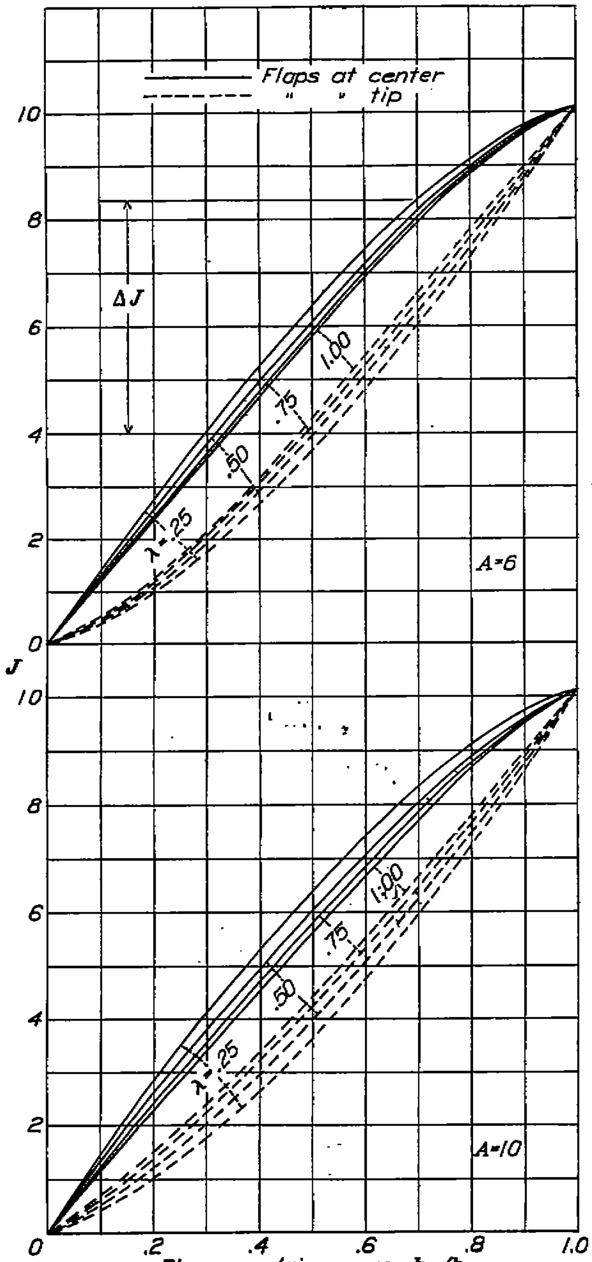

FIGURE 2.—Factor of angle of zero lift, J.
$\alpha_{s_{(L=0)}} = \alpha_{l_{0_s}} - J\Delta c_l$

The computed variation of the factor J with flap span is shown in figure 2 for various aspect ratios and taper ratios.

Although the values of J given in figure 2 apply specifically to wings in which the flap-chord ratio, or Δc_l, is constant along the portion with flaps and in which the flaps begin either at the center or at the tips, the results may be used to predict the angle of zero lift for any starting point of the flaps and for any Δc_l distribution as long as they are symmetrical about the wing center. For example, if flaps of uniform flap-chord ratio extend from 0.3b to 0.7b, the proper value of the factor J is the difference between the values for 0.3b and 0.7b as shown by ΔJ in figure 2. The extension to the case of a nonuniform symmetrical distribution of Δc_l consists simply in considering the resulting Δc_l distribution to be caused by a series of elemental flaps of various lengths and performing either a numerical or a graphical integration for the value of J. (See procedure given in reference 3.) In cases where the variation of Δc_l along the span is slight, however, the use of an average value of Δc_l is justified.

Lift-curve slope.—The wing lift-curve slope, a, per degree may be found from the equation

$$a = f\frac{\bar{a}_0}{1+\frac{57.3\bar{a}_0}{\pi A}} \tag{3}$$

where

A is wing aspect ratio, b^2/S.

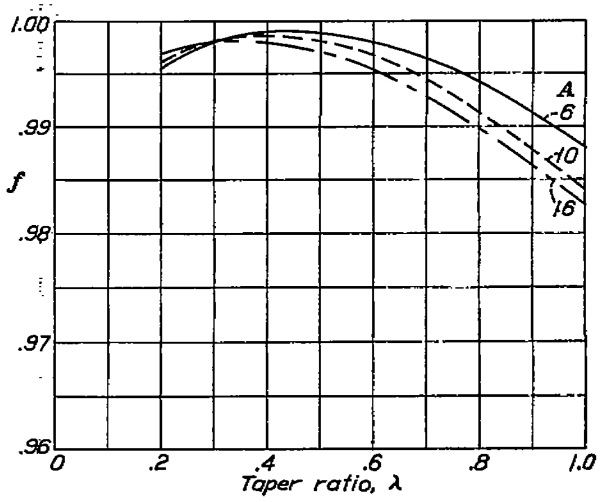

FIGURE 3.—Factor of wing lift-curve slope, f.
$a = f\frac{\bar{a}_0}{1+\frac{57.3\bar{a}_0}{\pi A}}$

f, a theoretical factor given in figure 3. This factor has been plotted from results given in reference 1.

$\bar{a}_0$, the weighted average of the section lift-curve slopes.

An average slope, weighted according to chord length, must be used because the slope of the sections with flaps may be considerably different from the slope of the sections without flaps. If the section lift-curve slopes are constant across the spans of the flapped and the unflapped parts of the wing, then $\bar{a}_0$ may be found in terms of the fraction of the area of the wing equipped with flaps:

$$\bar{a}_0=\frac{S_f}{S}a_{0_f}+\left(1-\frac{S_f}{S}\right)a_0 \tag{4}$$

where

a_0 is the lift-curve slope of section without flaps, per degree.

a_{0_f}, lift-curve slope of section with flaps, per degree.

S, area of wing.

S_f, area of part of wing equipped with flaps.

If a_0 and a_{0_f} are not constant across the two parts of the wing, then $\bar{a}_0$ may be found by integration.

namic centers is unaltered by deflecting the flaps, the x position of the wing aerodynamic center would be the same with the flaps either deflected or neutral.

If the aerodynamic center of the root section is taken as a reference point and the aerodynamic centers of all the wing sections are assumed to lie on a straight line making an angle Λ with the lateral axis (see fig. 4), then the x location of the load center is given by

$$x_{a.c.}=Hb\tan\Lambda \tag{5}$$

From equation (5), the aerodynamic-center position may also be related to the mean chord S/b and to the aspect ratio A by the equation

$$\frac{x_{a.c.}}{S/b}=HA\tan\Lambda \tag{6}$$

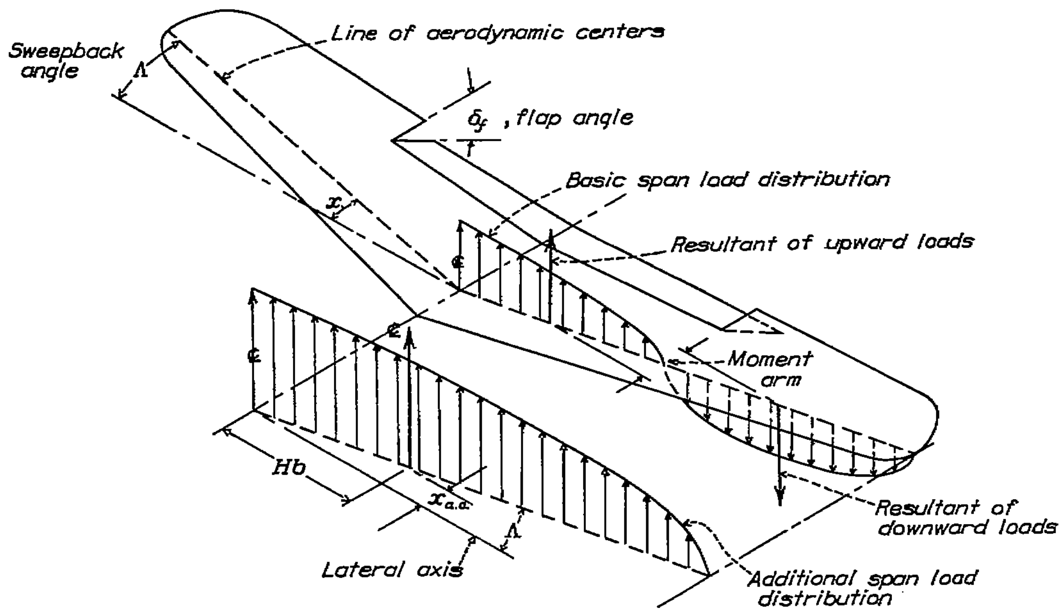

FIGURE 4.—Typical wing-flap combination showing basic and additional distributions.

Aerodynamic-center location.—The aerodynamic center of a wing is defined as the point about which the pitching moment is constant up to high lift coefficients; and, since the resultant lift must act through this point, the x position of the wing aerodynamic center is, in effect, nothing more than the fore-and-aft location of the centroid of the load distribution.

The load distribution of a wing with flaps is considered in this report to consist of the two components that are shown in figure 4. The basic load distribution is the span loading for zero lift with the flaps deflected; its ordinates are proportional to the value of Δc_l. The additional load distribution is that for the wing with flaps neutral; the total lift, however, is the same as that for the wing with flaps deflected. Since the basic load distribution contributes no lift, it does not enter into the determination of the aerodynamic center. Thus, if the chordwise position of the section aerody-

For a wing with sweepforward, Λ is negative and the aerodynamic center of the wing is ahead of the aerodynamic center of the root section.

Values of H are shown in figure 5.

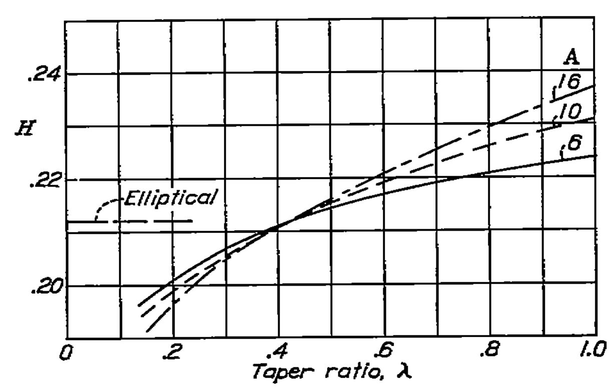

FIGURE 5.—Factor of wing aerodynamic center, H.
$\frac{x_{a.c.}}{S/b}=HA\tan\Lambda$

Pitching moment.—As shown in figure 4, the upward and the downward parts of the basic load form a couple having a magnitude that increases directly with the semispan length, the angle of sweepback, and the flap deflection, i. e., Δc_l. An equation for the pitching moment due to the basic load distribution can thus be written:

$$M_{l_b}=k\frac{b}{2}\Delta c_l \tan \Lambda\, qS$$

where k accounts for variations with wing taper, aspect ratio, and flap span. Because the basic load distribution is zero with no flap and is also zero with a full-span flap, the factor k would have a maximum value at an intermediate flap span.

Transforming the preceding equation into the coefficient form gives

$$C_{m_{l_b}}=G\Delta c_l\, A \tan \Lambda \tag{7}$$

where values of G are given in figure 6. For a wing with sweepback (Λ positive), the sign of the pitching-moment coefficient due to the basic lift is positive if the flap deflection introduces an effective washout toward the tip (e. g., flaps at the center deflected downward or flaps at the tip deflected upward). For a wing with sweepforward, the sign of $C_{m_{l_b}}$ is negative for the same flap deflections.

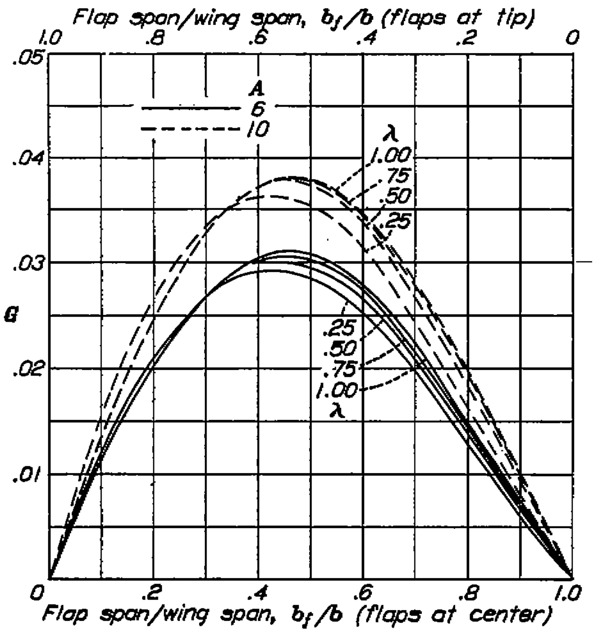

FIGURE 6.—Factor of basic lift pitching moment, G. $C_{m_{l_b}}=G\Delta c_l A \tan \Lambda$

In case the aerodynamic centers do not lie on a straight line so that the angle of sweepback is not constant along the span, $C_{m_{l_b}}$ may be graphically obtained from the equation

$$C_{m_{l_b}}=\frac{2b}{S^2}\int_0^{b/2} xc_{l_b}c\,dy \tag{8}$$

where, at any point along the span,

x is the moment arm measured from the aerodynamic center of the root section and parallel to the root chord (positive, rearward; negative, forward).

c_{l_b}, the section lift coefficient for the basic loading.

c, the chord.

In order to obtain the total pitching-moment coefficient, that due to the sections must also be added. This pitching-moment coefficient is given by

$$C_{m_S}=\frac{2b}{S^2}\int_0^{b/2} c_{m_{a.c.}}c^2dy \tag{9}$$

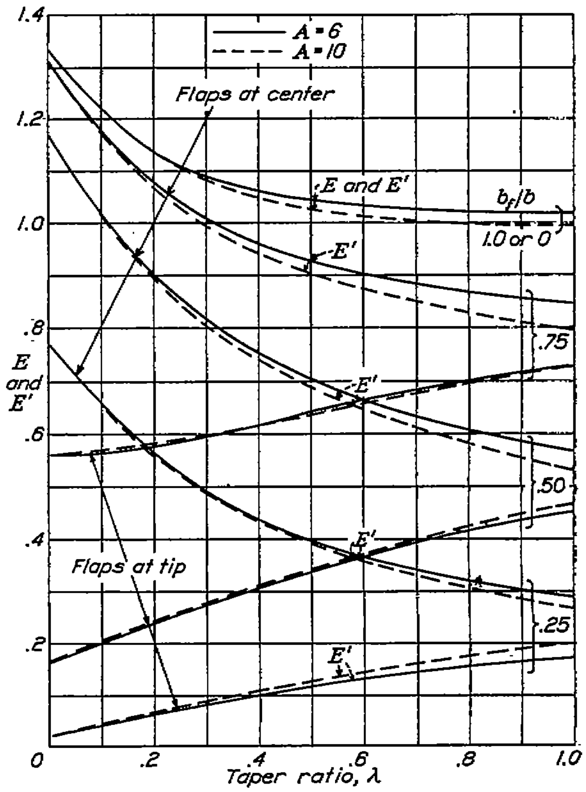

FIGURE 7.—Factors of section pitching moment, E and E'. $C_{m_S}=Ec_{m_0}+E'\Delta c_m$

For wings with flaps, however, the value of the section pitching-moment coefficient $c_{m_{a.c.}}$ may be assumed to consist of two parts: One denoted by c_{m_0}, the section coefficient with flaps neutral; and the other denoted by Δc_m, the increase in the section coefficient above c_{m_0} due to the flaps. If c_{m_0} is constant across the span and Δc_m is constant across the flap span (i. e., the flap-chord ratio is constant), then the pitching-moment coefficient due to the sections can be given by

$$C_{m_S}=Ec_{m_0}+E'\Delta c_m \tag{10}$$

Values of E and E' for these conditions are given in figure 7 for the tapered wings. These values have been determined from the relations

$$E=\frac{2b}{S^2}\int_0^{b/2} c^2dy$$

$$E'=\frac{2b}{S^2}\int_{y_f}^{b_f/2} c^2dy$$

If neither c_{m_0} nor Δc_m were constant across the span, then it would be necessary to use equation (9) and to evaluate C_{m_S} by an integration, as will be illustrated later. The total wing pitching-moment coefficient is given by

$$C_{m_{a.c.}}=C_{m_S}+C_{m_{l_b}} \tag{11}$$

The coefficient $C_{m_{a.c.}}$ is defined by the equation

$$M=C_{m_{a.c.}}q\frac{S^2}{b} \tag{12}$$

where M is the total pitching moment.

Induced drag.—For any wing with a twist that is symmetrical about the wing center line, the induced-drag coefficient may be given by the equation

$$C_{D_i}=\frac{C_L^2}{\pi A u}+C_L\Delta c_l v+\Delta c_l^2 w \tag{13}$$

The factors u, v, and w for wings with partial-span flaps, i. e., for the case of an abrupt twist, are given in figure 8.

The first term on the right-hand side of equation (13) is the usual induced-drag coefficient of an untwisted wing and the other two terms result from the aerodynamic wing twist introduced by deflecting the flaps. It can be seen from figure 8 that, for certain taper ratios, the v and the w factors are of opposite sign and their contributions counteract each other. In fact, under certain conditions, the sum of the last two terms may be slightly negative; and, as a result, the elliptical wing induced-drag coefficient may be approached. This tendency exists when the flaps are so placed and deflected that an elliptical loading is approximated.

EXPERIMENTAL RESULTS

APPARATUS AND TESTS

In order to provide a check on the reliability of the theoretical factors that have been presented, two tapered wings with partial-span flaps were tested. In addition, tests were made of three rectangular wings with full-span flaps to provide section data for use in calculating the characteristics of the tapered wings. The wings were made of aluminum alloy and had an area of 150 square inches.

A list of the tapered wings and the different flap lengths used is given in table I, together with the taper ratio, the aspect ratio, and the airfoil sections of the root and the construction tip (the extreme tip). The tips were rounded as shown in figure 1. The N. A. C. A. 23012 tapered wing had a moderate sweepback (line through quarter-chord points) but the N. A. C. A. 5–10–16 tapered wing had no sweepback. In the construction of the wings, straight-line elements were used between corresponding points of the root and the construction tip sections. For the N. A. C. A. 23012 wing, the chords of all sections along the span were in one plane; whereas, for the N. A. C. A. 5–10–16 wing, the highest points of the upper surface of each section were in one plane. The ordinates of the N. A. C. A. 5–10–16 wing are given in reference 4.

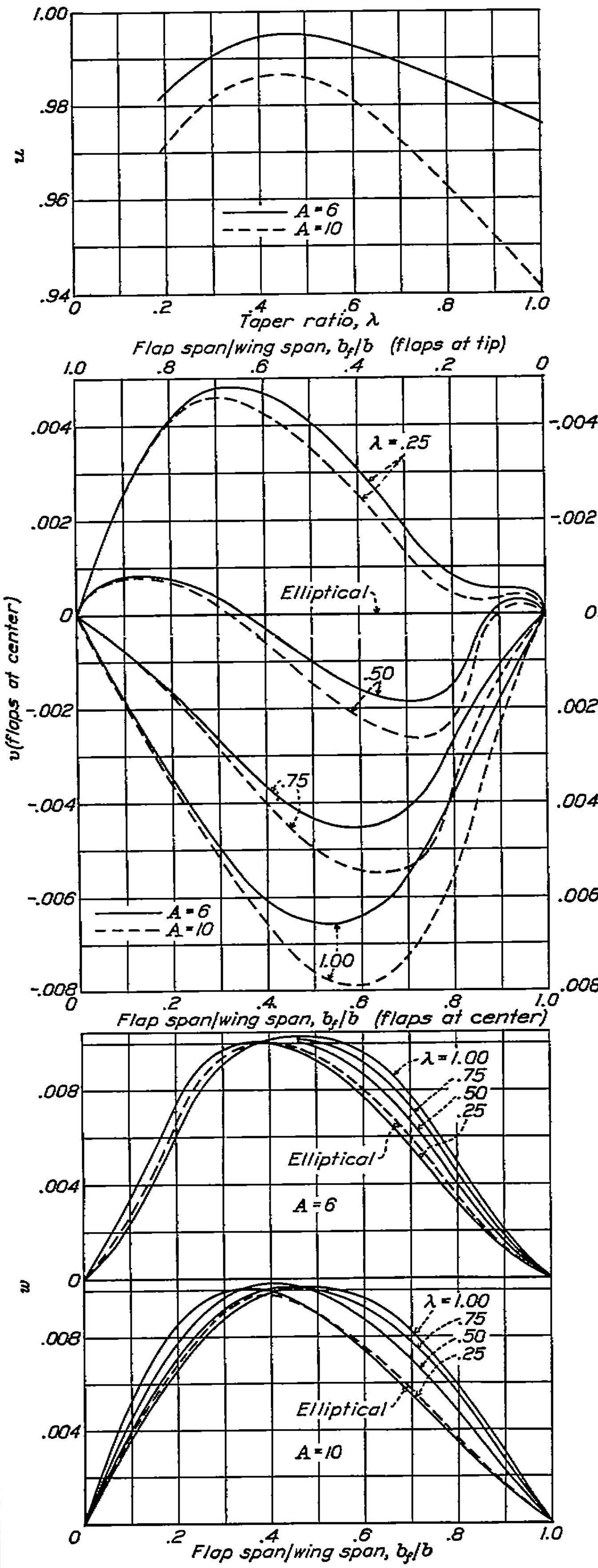

FIGURE 8.—Factors of induced drag, u, v, and w.

$$C_{D_i}=\frac{C_L^2}{\pi A u}+C_L\Delta c_l v+\Delta c_l^2 w$$

The rectangular wings had N. A. C. A. 23009, 23012, and 23015 sections and were included to provide airfoil section characteristics to aid in calculating the characteristics of the tapered wings.

Plain 0.2c flaps deflected downward 20° were built nto all the wings and were made to simulate flaps pivoted about the midpoint of the thickness at 0.8c. Fillets of small radii were used to join the flap to the wing and to seal the gap, as indicated at the top of figure 9.

All the wings were tested in the variable-density wind tunnel at a pressure of 20 atmospheres. The lift, the drag, and the pitching moment were measured at $c_{m_{(a.c.)_0}}$ are given about the aerodynamic-center position with the flap neutral.

The results of the tests of the tapered wings are presented in the usual manner in figures 12 to 17. In addition to the usual characteristics, the lift-curve peaks are given for two values of the effective Reynolds Number to indicate the scale effect on $C_{L_{max}}$. The Reynolds Number is based on the mean chord S/b. On the right side of the figures, effective profile-drag coefficients are given. This coefficient is the total drag coefficient with the induced-drag coefficient for elliptical span loading deducted, that is, $C_{D_e}=C_D-C_L^2/\pi A$. The values of C_{D_e} have been corrected to effective

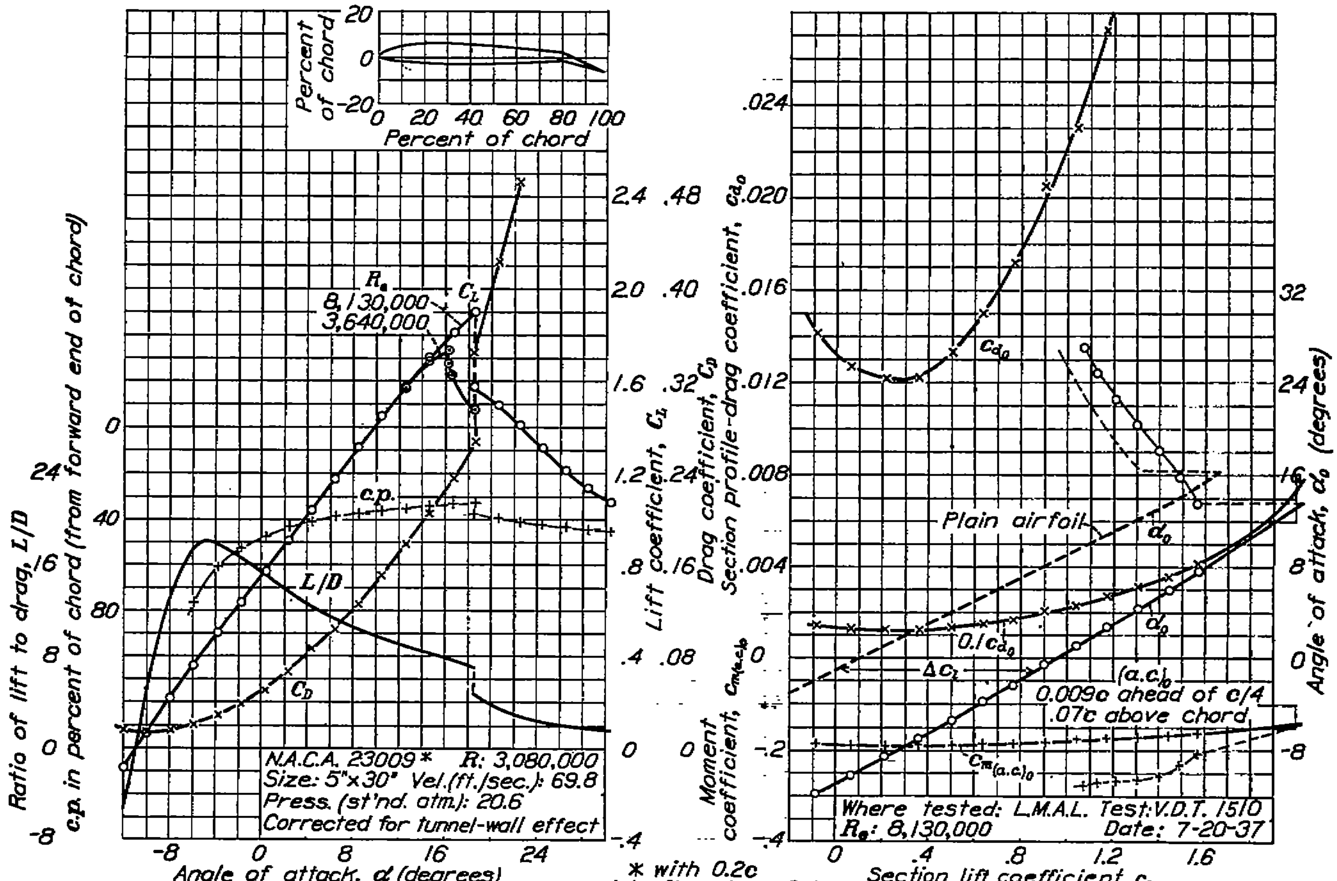

FIGURE 9.—The N. A. C. A. 23009 airfoil with 0.2c plain flap down 20°.

the usual high Reynolds Number and, in addition, the maximum lift was measured at a lower Reynolds Number to indicate the scale effect on $C_{L_{max}}$. The method of making and correcting the tests and a description of the tunnel are given in reference 5.

The results of the tests are presented in the usual form as figures 9 to 17. The results of the tests of rectangular wings, plotted on the left side of figures 9 to 11, have been corrected to aspect ratio 6; whereas the results given on the right side have been corrected to airfoil section characteristics by the method explained in reference 6. The pitching-moment coefficients Reynolds Number R_e by subtracting an increment (0.0011) to allow for the reduction in skin-friction drag when extrapolating from test to effective Reynolds Number (reference 6). The pitching-moment coefficients given are based on the mean chord S/b so that $C_m=M/qS(S/b)=Mb/qS^2$. The coefficients for each wing-and-flap combination are given about an axis through the aerodynamic center determined by the method given in the appendix of reference 4. The location of the aerodynamic center given in the upper part of the right side of the figures is measured from the quarter-chord point of the root chord and is in terms of S/b.

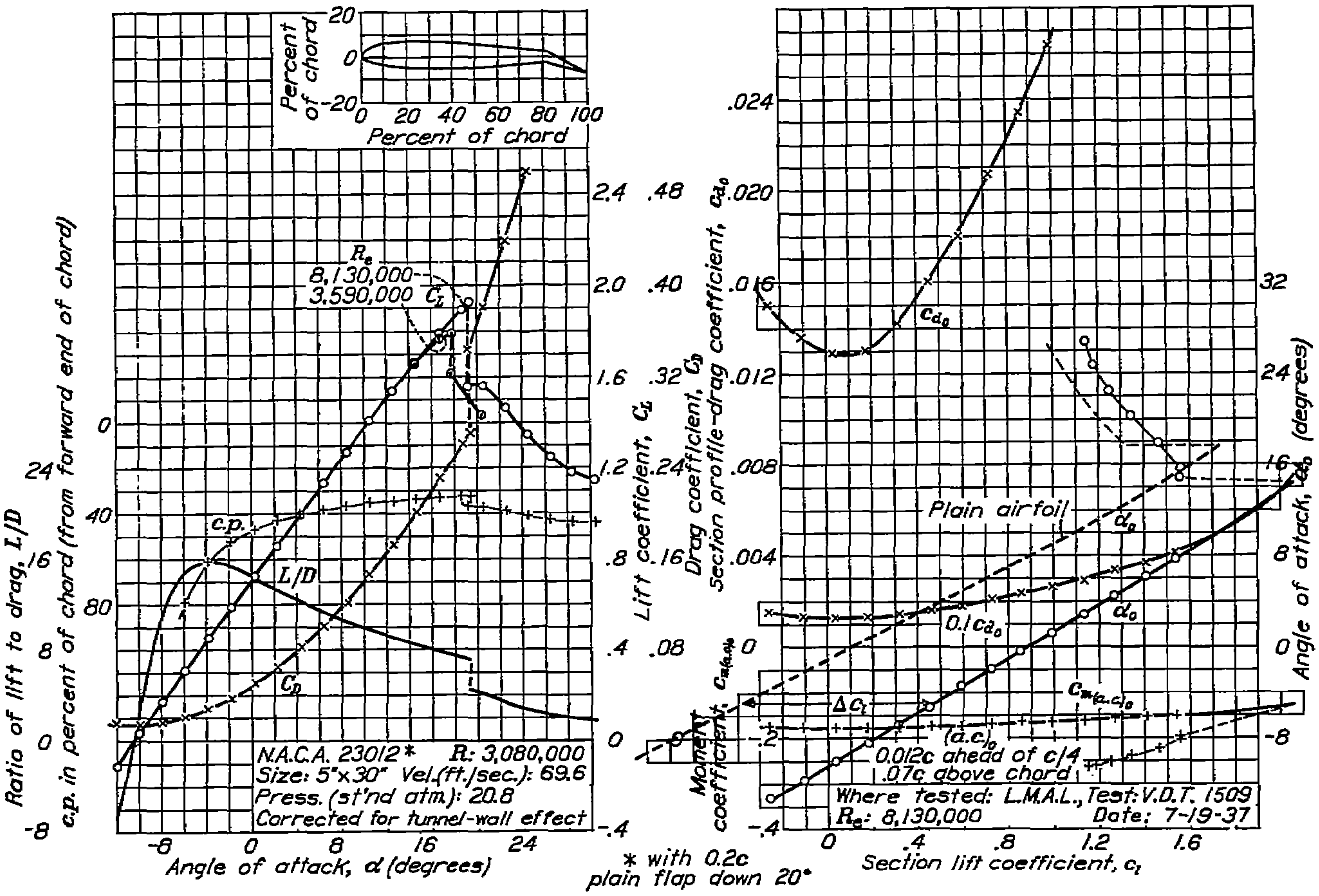

FIGURE 10.—The N. A. C. A. 23012 airfoil with 0.2c plain flap down 20°.

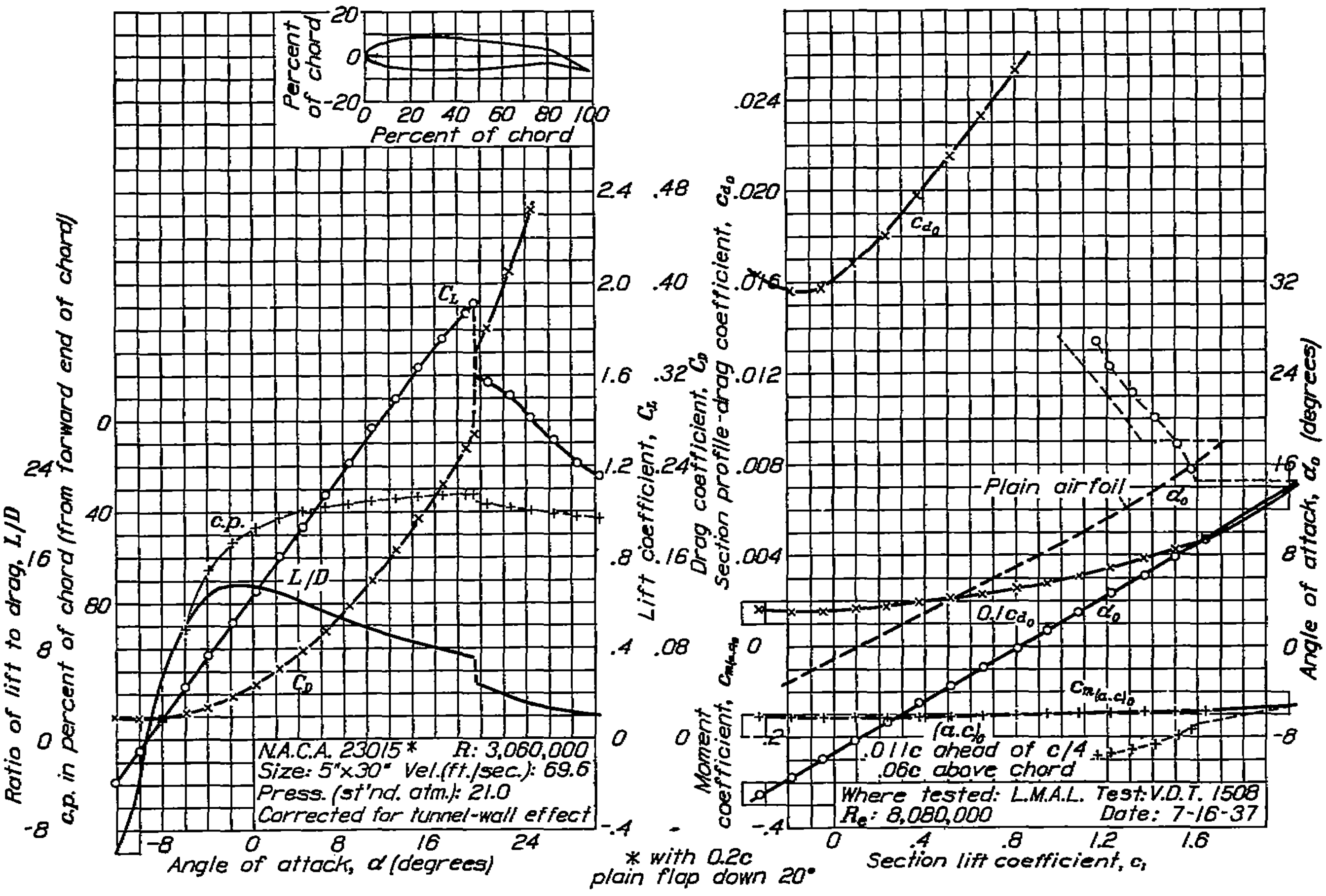

FIGURE 11.—The N. A. C. A. 23015 airfoil with 0.2c plain flap down 20°.

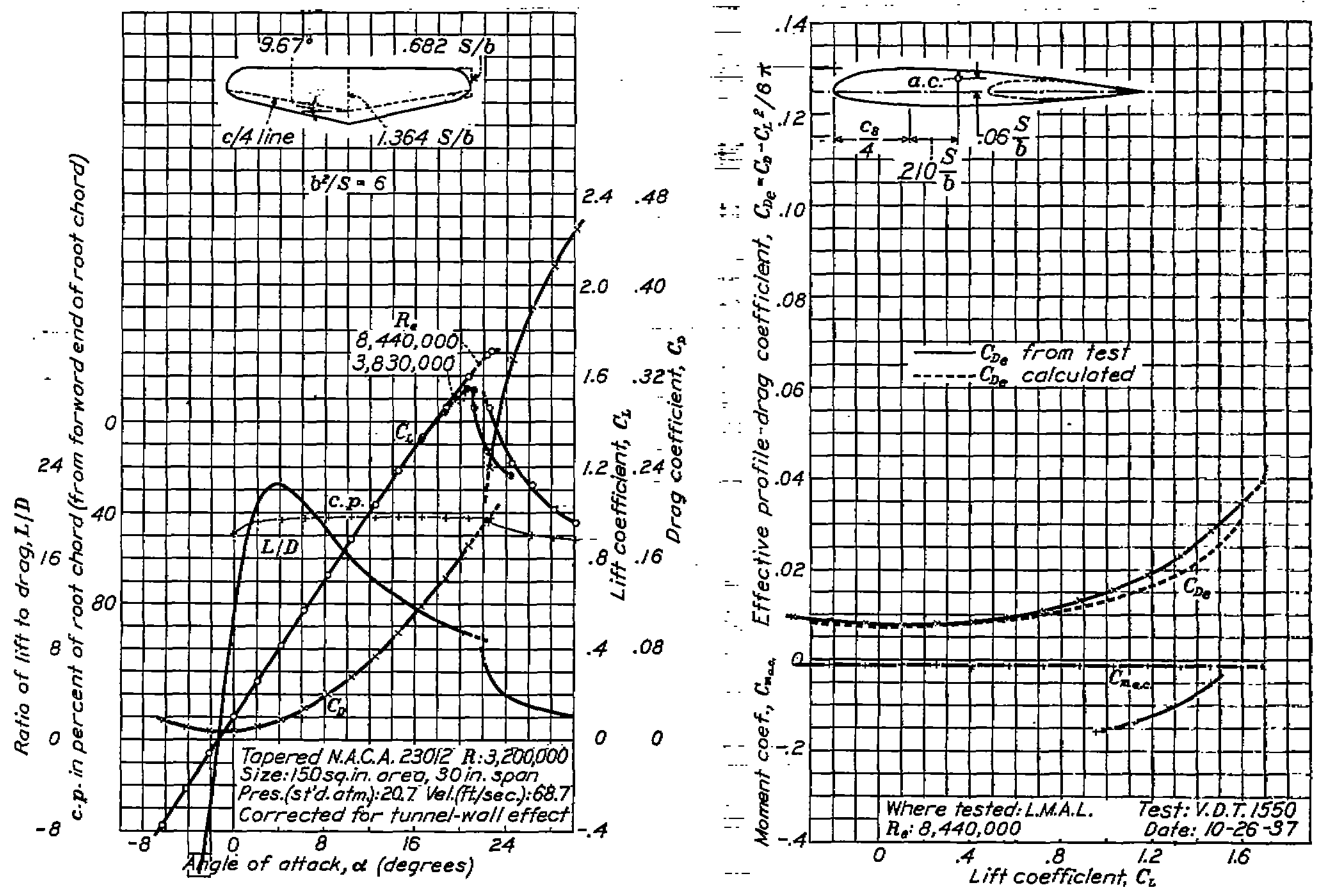

FIGURE 12.—The tapered N. A. C. A. 23012 airfoil.

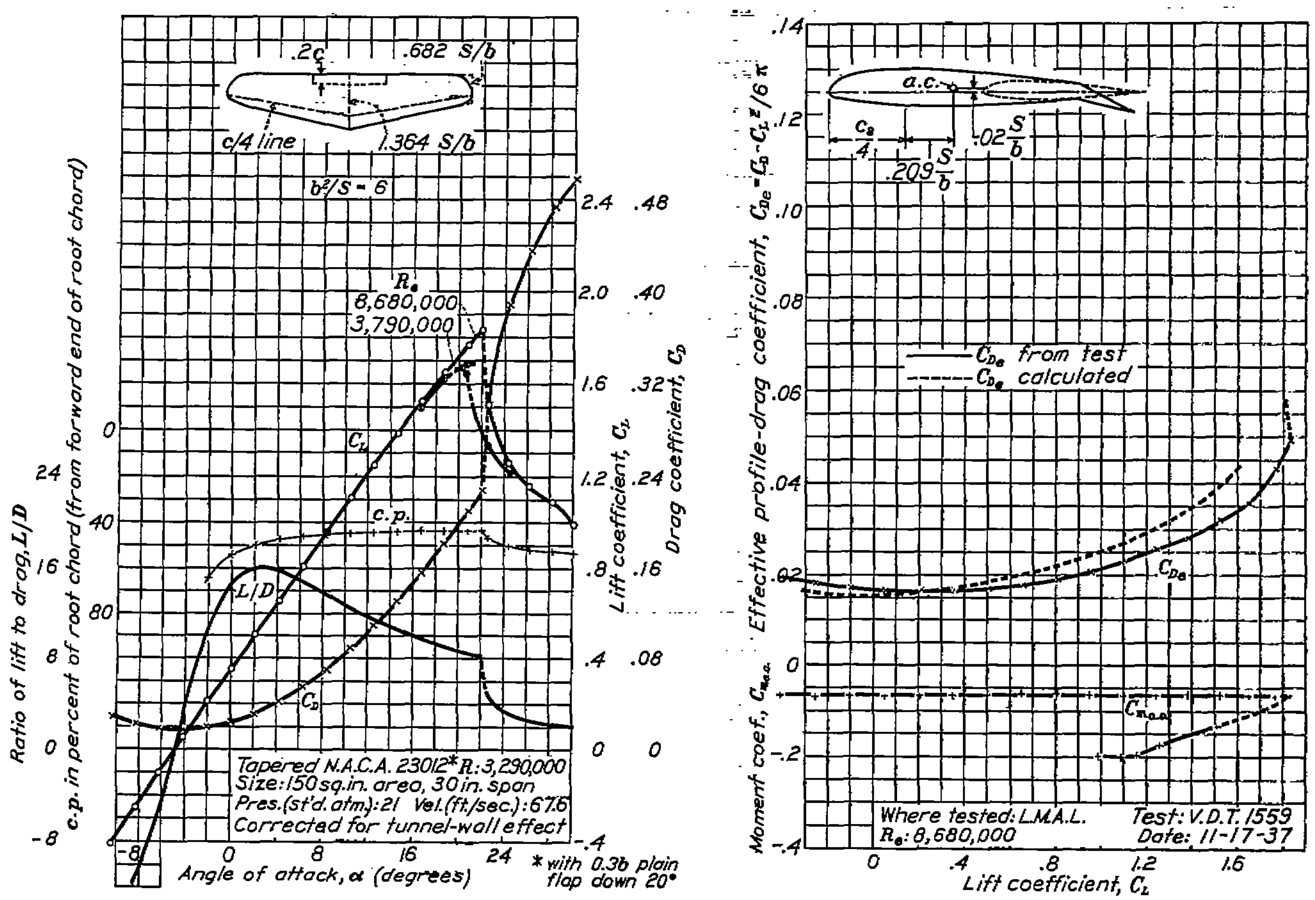

FIGURE 13.—The tapered N. A. C. A. 23012 airfoil with 0.3b plain flap down 20°.

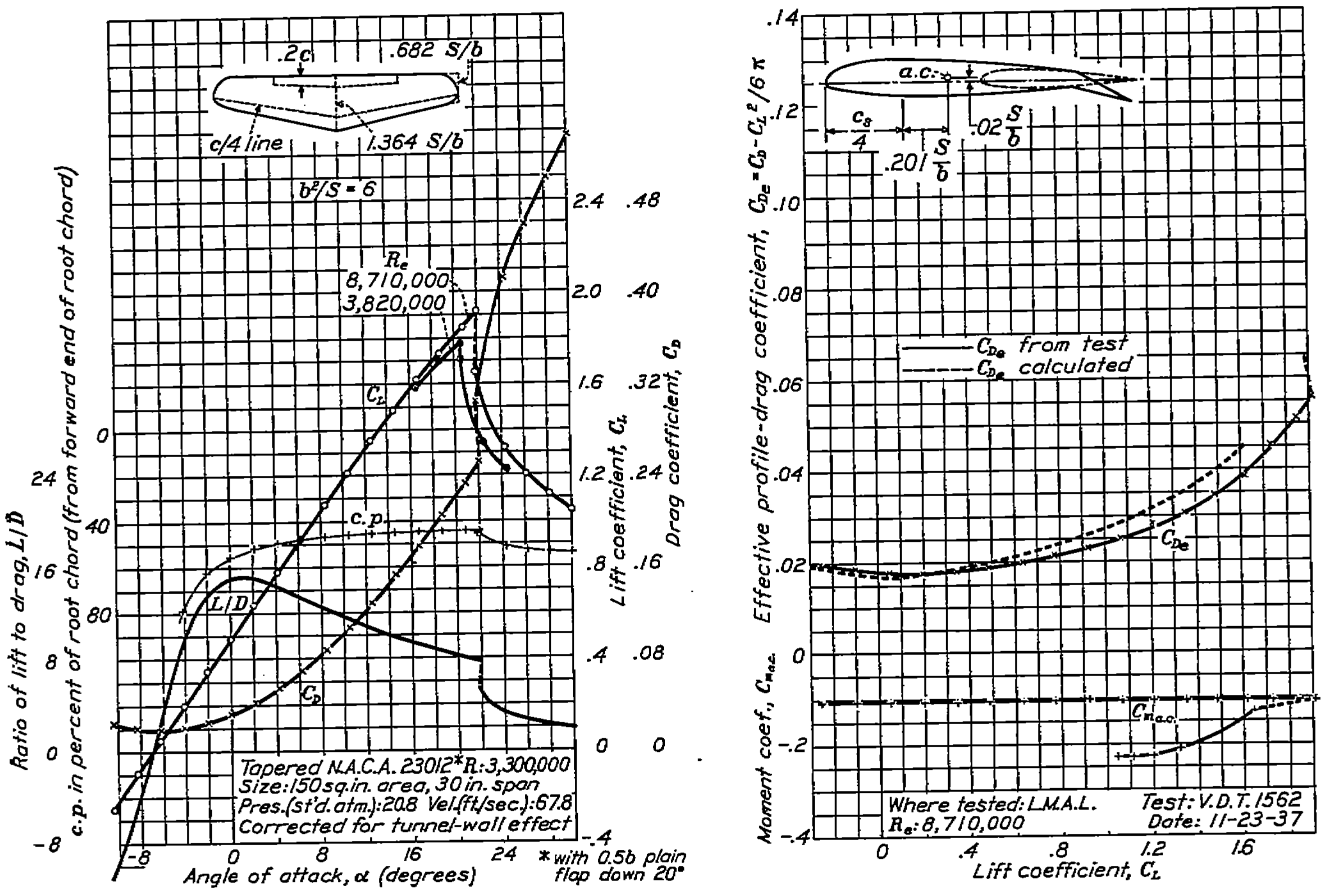

FIGURE 14.—The tapered N. A. C. A. 23012 airfoil with 0.5b plain flap down 20°.

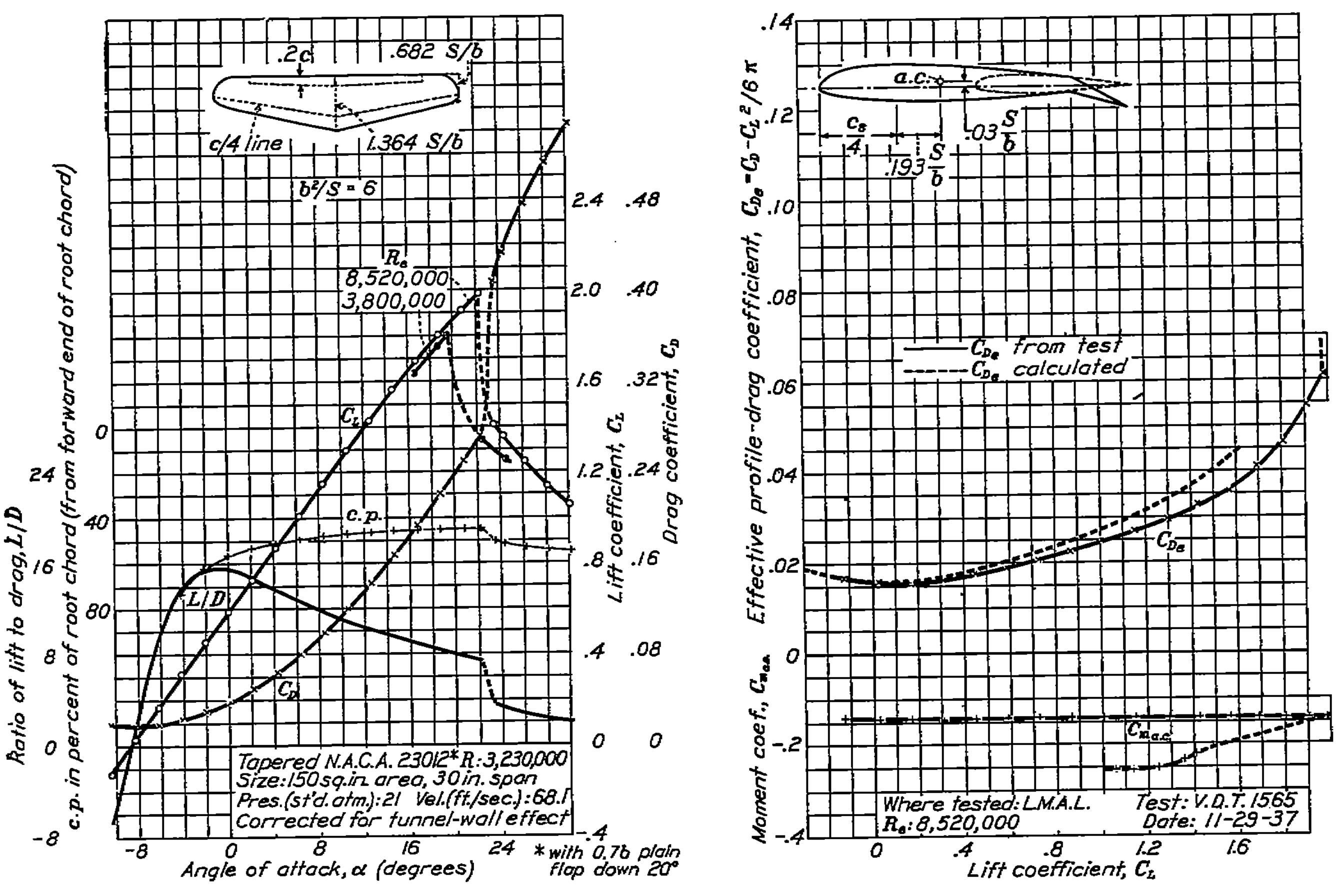

FIGURE 15.—The tapered N. A. C. A. 23012 airfoil with 0.7b plain flap down 20°.

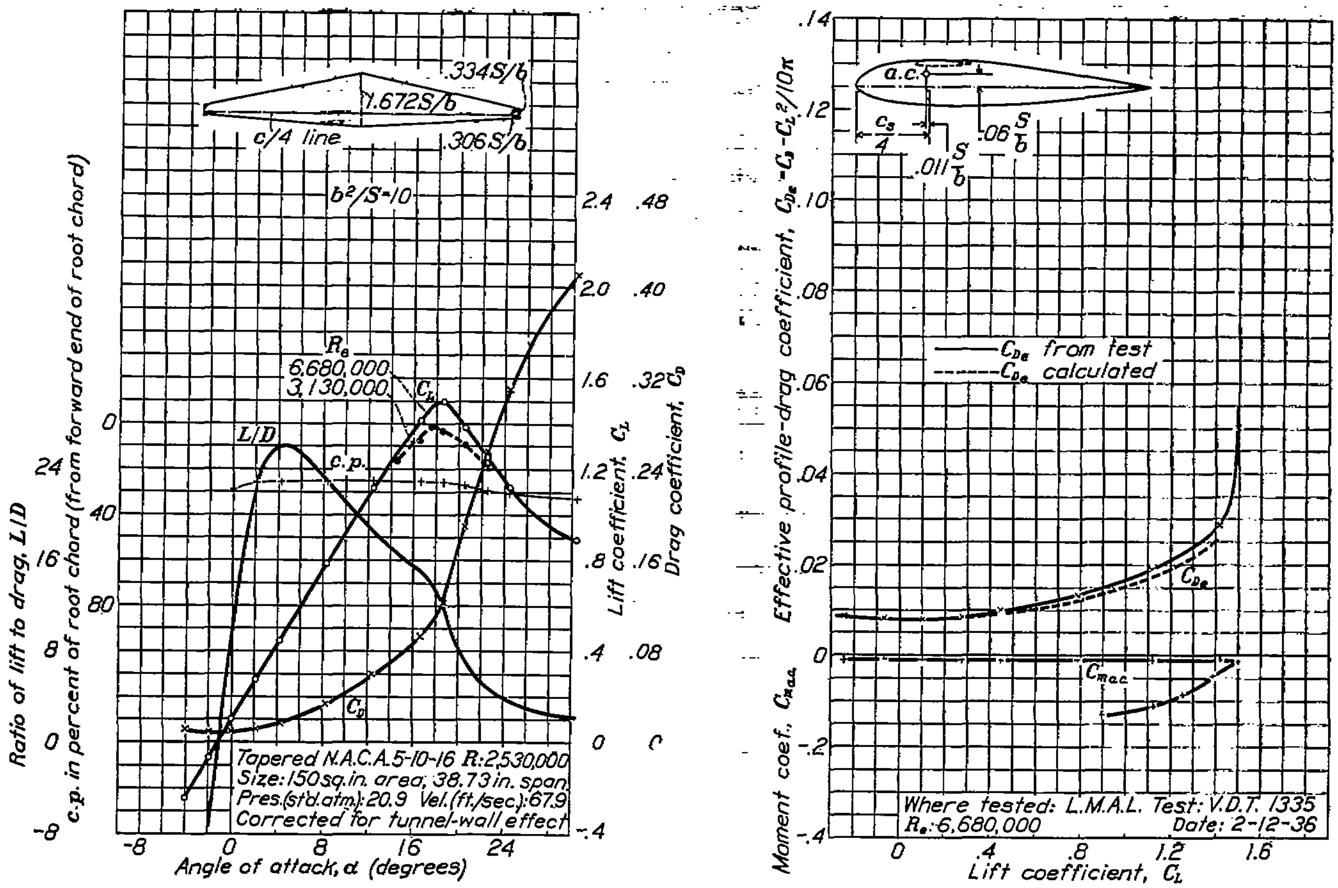

FIGURE 16.—The tapered N. A. C. A. 5-10-16 airfoil.

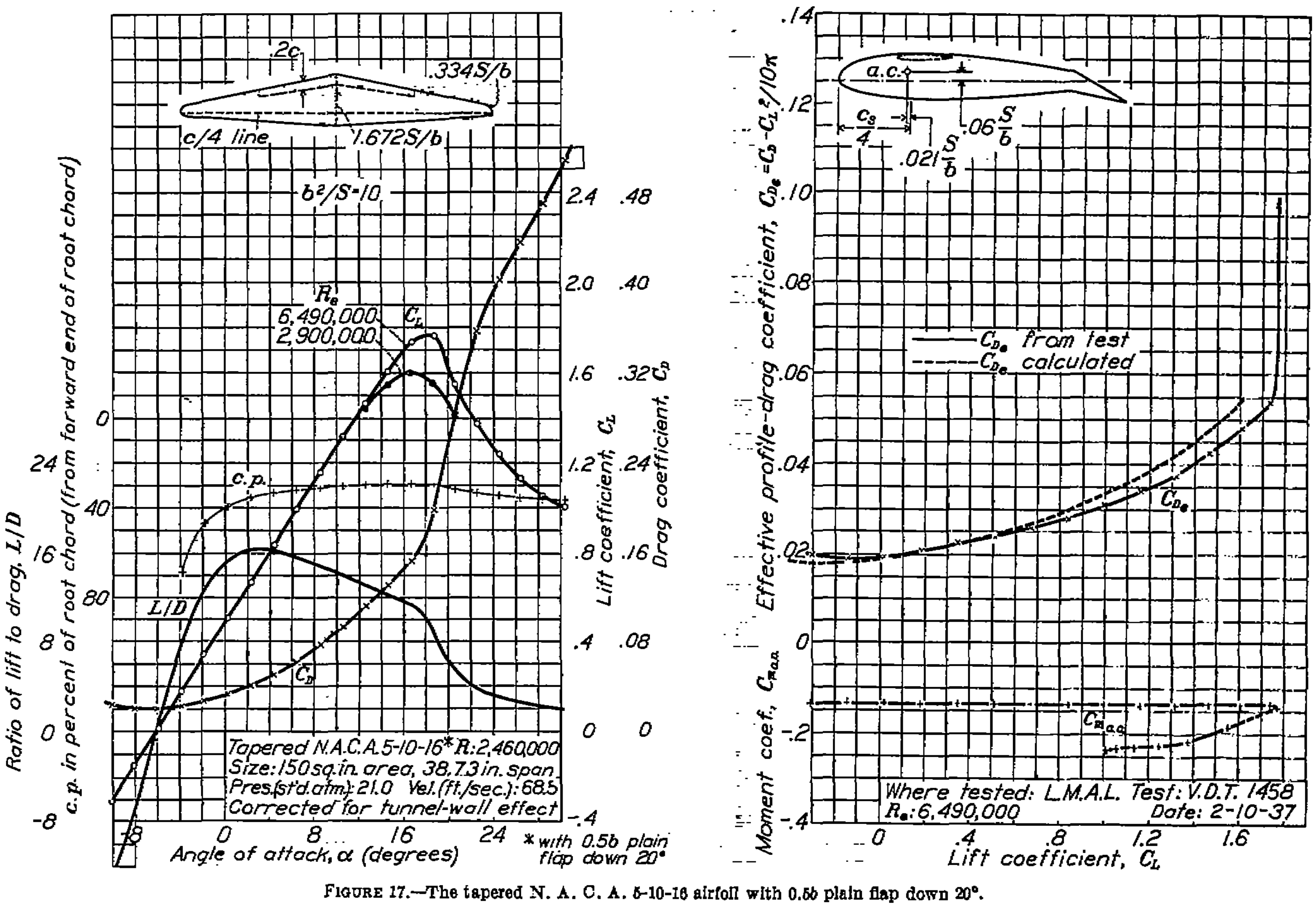

FIGURE 17.—The tapered N. A. C. A. 5-10-16 airfoil with 0.5b plain flap down 20°.

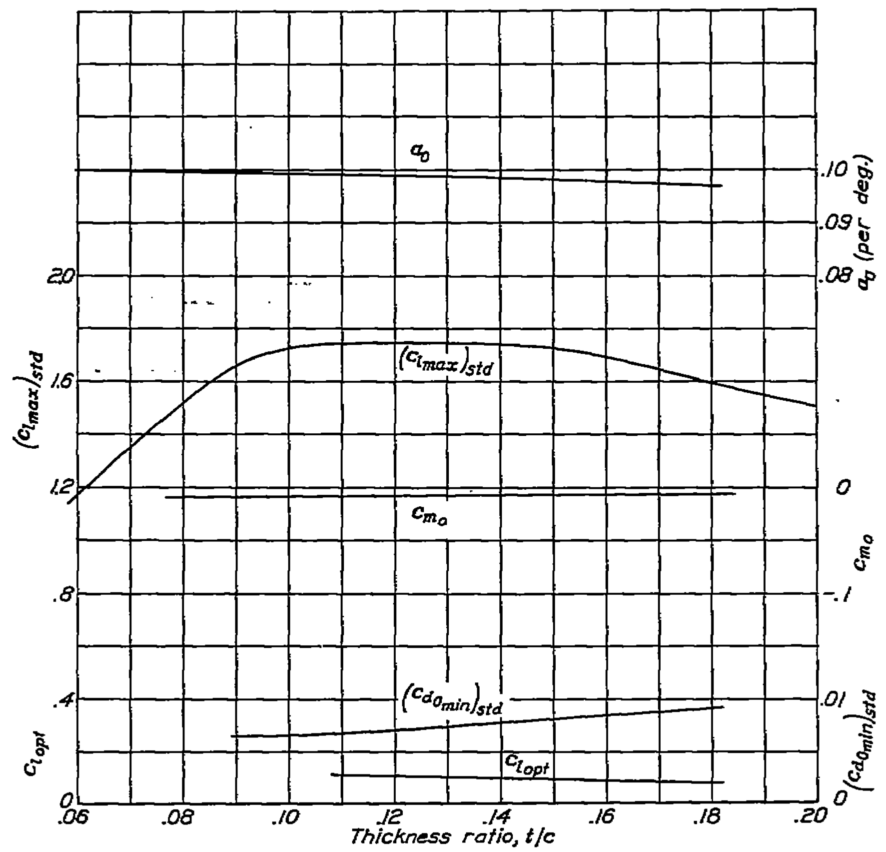

FIGURE 18.—Variation of section data with thickness. The N. A. C. A. 230 series airfoils; effective Reynolds Number, 8,200,000.

CALCULATED CHARACTERISTICS OF THE WINGS

The factors previously presented were applied to the calculation of the characteristics of the wings used in the tests and the results are summarized in table I. The calculations will be illustrated for the tapered N. A. C. A. 23012 wing with the 0.5*b* flap.

Angle of zero lift and lift-curve slope.—The angle of zero lift by equation (2) is:

$$\alpha_{s_{(L=0)}}=-1.2-(6.07\times0.90)=-6.7°$$

The value of Δc_l (0.90) was measured from figure 10 at approximately the average lift coefficient of the basic c_{l_b} distribution of the flapped portion of the wing. The average lift coefficient was estimated from column 15 of table II.

The lift-curve slope was calculated from equation (3), the value of *f* being taken from figure 3:

$$a=0.999\frac{0.091}{1+\frac{57.3\times0.091}{\pi\times6}}=0.071$$

The value of $\bar{a}_0$ (equation (4)), values of a_0 and a_{0_f} having been taken from figures 18 and 19, is

$$\bar{a}_0=\frac{89.6}{150}\times0.085+\left(1-\frac{89.6}{150}\right)\times0.099=0.091$$

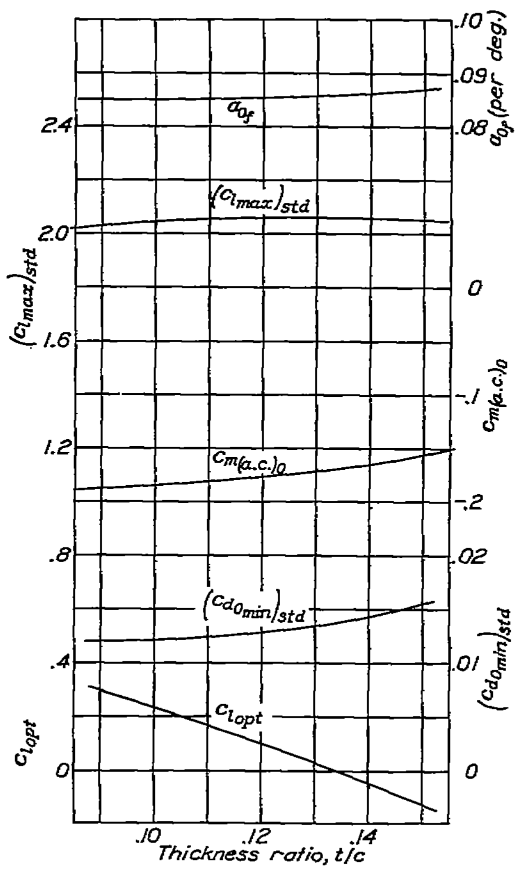

FIGURE 19.—Variation of section data with thickness. The N. A. C. A. 230 series airfoils with 0.2c plain flap down 20°; effective Reynolds Number, 8,200,000.

Aerodynamic center and pitching moment.—The x position was found directly from equation (6):

$$\frac{x_{a.c.}}{S/b}=HA\tan\Lambda=0.214\times6\times0.1703=0.219$$

The appropriate values of Λ and H (fig. 5) were used and the aerodynamic centers of the wing sections were considered to be at the quarter-chord points.

The pitching-moment coefficient C_{m_S} due to the moments of the airfoil sections (equation (9)) was obtained, as shown in figure 20, from the area above the $c_{m_{a.c.}}c^2$ curve. Figure 20 illustrates the general method that may be applied to any plan form and any distribution of $c_{m_{a.c.}}$ across the span. The values of the necessary pitching-moment coefficients were taken from figures 18 and 19 and are for a value of c_l of zero, in which case $c_{m_{a.c.}}=c_{m_{(a.c.)_0}}$. If $c_{m_{(a.c.)_0}}$ varies appreciably with c_l, it should be taken at the average value of c_l over the portion of the wing with flaps. In this example, C_{m_S} could also have been calculated from the values of E and E' given in figure 7 because the increment, as well as the initial value of the pitching-moment coefficient, was substantially constant across both the flapped and the unflapped parts of the span.

The pitching-moment coefficient due to the basic lift distribution is given by formula (7)

$$C_{m_{l_b}}=G\Delta c_l A\tan\Lambda=0.029\times0.77\times6\times0.1703=0.023$$

The value of G was taken from figure 6 and the value of Δc_l was taken at an intermediate c_l ($c_l=1.0$) from figure 10 for the N. A. C. A. 23012 airfoil with flap. Although Δc_l varies with the c_l at which it is taken, the exact value used does not affect the value of $C_{m_{l_b}}$ appreciably unless the sweepback is large. When the quarter-chord points do not lie on a straight line so that the angle of sweepback cannot be measured, $C_{m_{l_b}}$ may be computed from formula (8). The total pitching-moment coefficient about the axis through the aerodynamic center is then

$$C_{m_{a.c.}}=C_{m_S}+C_{m_{l_b}}=-0.128+0.023=-0.105$$

Drag.—The induced drag was calculated from formula (13) using values of u, v, and w from figure 8 and a value of Δc_l at an intermediate value of c_l ($c_l=1.0$) for the N. A. C. A. 23012 airfoil with flap (fig. 10). Thus C_{D_i} for any value of C_L is

$$C_{D_i}=\frac{C_L^2}{\pi A\times0.986}+(-0.0010)0.77C_L+0.0100(0.77)^2$$

Values of C_{D_i} were calculated for a series of values of C_L.

The profile-drag coefficient of the wings was calculated by an integration of the section profile drag along the semispan as given by

$$C_{D_0}=\frac{b}{S}\int_0^1 c_{d_0}c\,d\left(\frac{y}{b/2}\right)$$

This integration has been graphically performed as shown in figure 21. The value of c_{d_0} at any point will, of course, depend upon the airfoil section, the lift coefficient, and the Reynolds Number at that point. The calculations are illustrated in table II for a C_L of 0.8 and follow the method of reference 4.

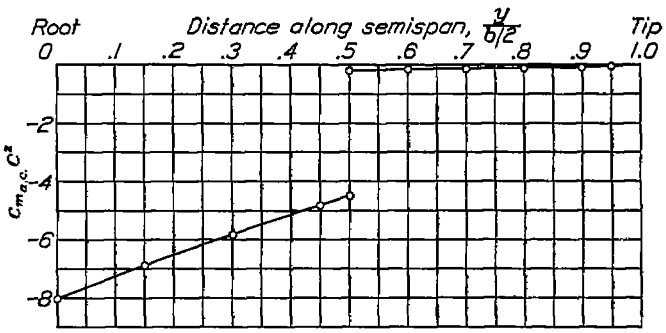

FIGURE 20.—Graphical determination of C_{m_S} for the N. A. C. A. 23012 airfoil with 0.55 plain flap.

$$C_{m_S}=\frac{b^2}{S^2}\int_0^1 c_{m_{a.c.}}c^2d\left(\frac{y}{b/2}\right)=-0.128$$

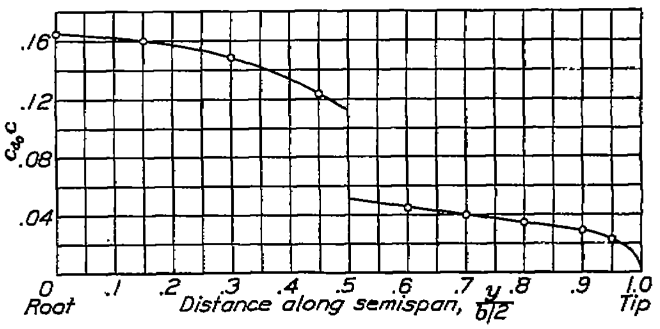

FIGURE 21.—Graphical determination of C_{D_0} for the N. A. C. A. 23012 tapered airfoil with 0.55 plain flap.

$$C_{D_0}=\frac{b}{S}\int_0^1 c_{d_0}c\,d\left(\frac{y}{b/2}\right)=0.0184$$

Values of $c_{d_0}c$ were calculated at intervals along the semispan using the known lift distributions. The values of $(c_{l_{max}})_{std}$, $(c_{d_{0_{min}}})_{std}$, $c_{l_{opt}}$, and $\Delta c_{l_{max}}$ that appear in the various columns of table II were obtained from figures 18, 19, and 22. Section data for other flap deflections may be found in references 6 and 7. The values of $c_{d_{0_{min}}}$ were extrapolated to the Reynolds Number of each point along the semispan by the method given in figure 23 of reference 4. Although the formula given in reference 4 was derived for sections with moderate camber, it should apply approximately to airfoils with flaps.

The c_l distribution for $C_L=0.8$ was obtained from the equation

$$c_l=C_Lc_{l_{a1}}+c_{l_b}$$

where

$$c_{l_{a1}}=\frac{S}{cb}L_a \text{ and } c_{l_b}=\frac{\Delta c_l S}{cb}L_b$$

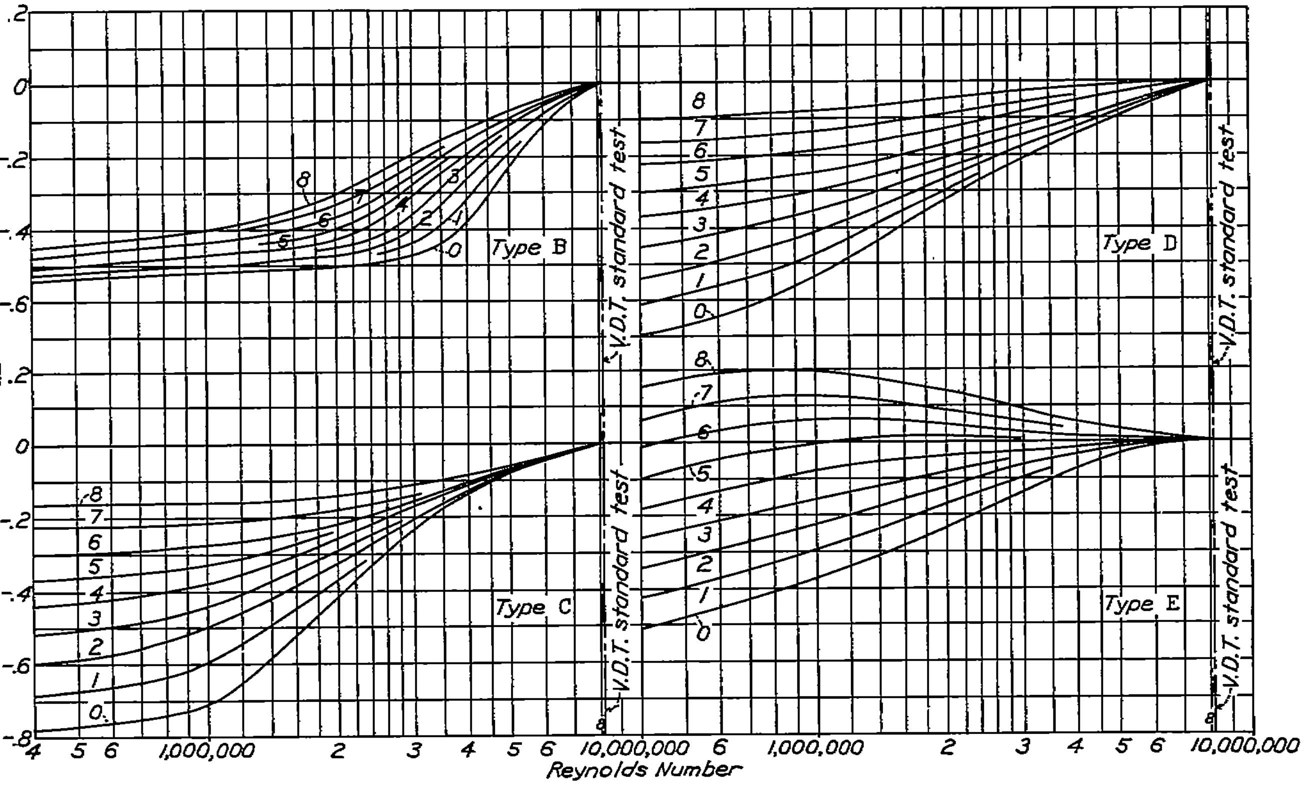

FIGURE 22.—Scale-effect corrections for $c_{l_{max}}$. In order to obtain the section maximum lift coefficient at the desired Reynolds Number, apply to the standard-test value the increment indicated by the curve that corresponds to the scale-effect designation (types B, C, D, or E) of the airfoil. (See reference 6, p. 32 and table II.)

The values of L_a and L_b at $b_f/b=0.5$ were obtained from reference 2 by cross-plotting against b_f/b. The value of Δc_l at $c_l=1.0$ for the flapped section was obtained from figure 10.

The profile-drag coefficient is given by the equation

$$c_{d_0}=c_{d_{0_{min}}}+\Delta c_{d_0}$$

where Δc_{d_0} depends on the quantity in column 19 of table II. Values of Δc_{d_0} were obtained from figure 23 for the sections without flaps (data taken from reference 6) and from figure 24 for the N. A. C. A. 23012 section with flap.

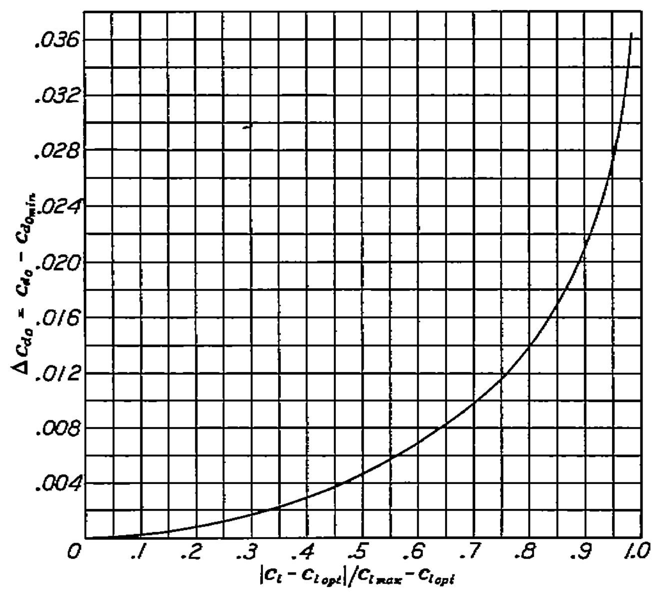

FIGURE 23.—Generalized variation of Δc_{d_0} for airfoil sections with flaps neutral.

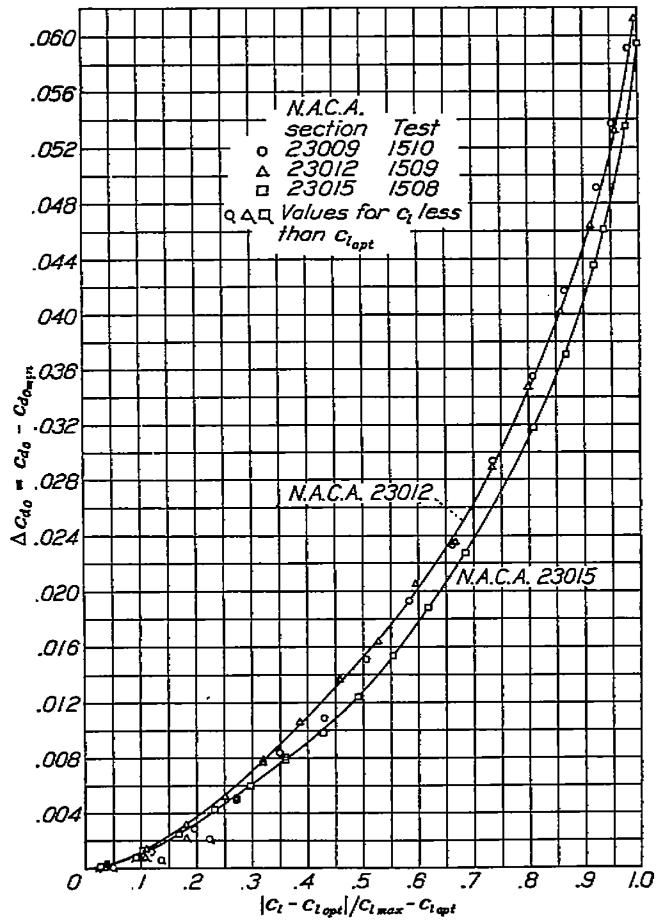

FIGURE 24.—Curves of Δc_{d_0} for airfoil sections with 0.2c plain flaps down 20°.

The values of $c_{d_0}c$ given in column 22 of table II were plotted in figure 21 and extrapolated to the flap end. From the area under the curve, C_{D_0} was found to be 0.0184. The process was repeated for other lift coefficients and for other wing-and-flap combinations and the results are plotted in figures 12 to 17 as effective profile-drag coefficients

$$C_{D_e}=C_{D_0}+C_{D_i}-C_L^2/\pi A$$

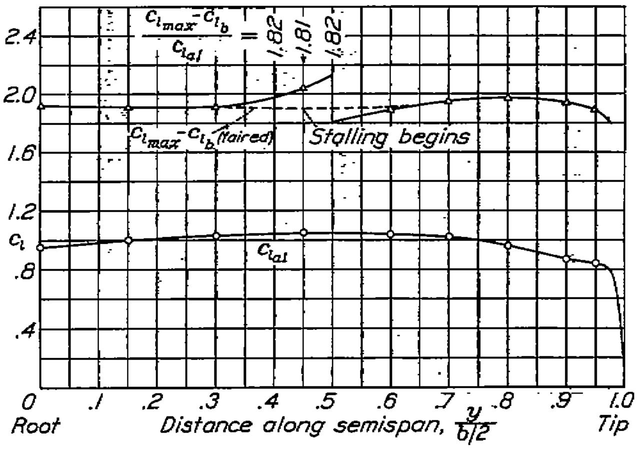

FIGURE 25.—Calculation of the C_L at which the N. A. C. A. 23012 airfoil with 0.5b plain flap begins to stall.

and the value of $C_{L_{max}}$ as indicated on figure 25. The dashed curve faired through the flap end was used. The solid line, which passes through calculated values of $c_{l_{max}}-c_{l_b}$, would have indicated stalling at the plain section just outboard of the flap end at a low $C_{L_{max}}$. Observations of the action of tufts, however, indicate that stalling does not necessarily begin at this point. It appears to be preferable to fair $c_{l_{max}}-c_{l_b}$ through the flap end, as shown. The calculated $C_{L_{max}}$ value is then higher and in better agreement with the test value. Tuft observations of the wings with 0.3b and 0.7b flaps indicated that stalling began at a point other than the predicted point so that the method can be expected to give only a rough indication of $C_{L_{max}}$.

COMPARISON OF THE CALCULATED AND THE EXPERIMENTAL RESULTS

The calculated and the experimental results are compared in table I. The angles of zero lift and the lift-curve slopes are in good agreement. The x positions of the aerodynamic center are in fair agreement although the experimental aerodynamic-center positions move more and more ahead of the calculated position as the flap length is increased, probably owing to the forward movement of the position of the aerodynamic center

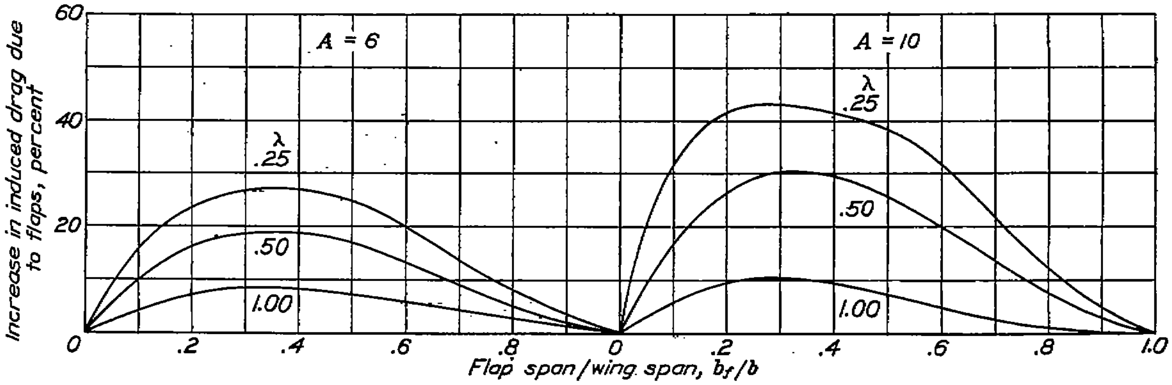

FIGURE 26.—Increase in induced drag due to the addition of flaps of various lengths. C_L, 1.0; Δc_l, 1.0.

Maximum lift coefficient.—The lift coefficients at which the wings should begin to stall were estimated by the method used in reference 1 except that, instead of plotting c_l for a series of values of C_L, the point at which which a section lift coefficient reaches $c_{l_{max}}$ (c_l curve becomes tangent to the $c_{l_{max}}$ curve) was found more conveniently by first deducting c_{l_b} from $c_{l_{max}}$ as in figure 25. (See column 23, table II.) The point at which the $c_{l_{a1}}$ curve (c_{l_a} for $C_L=1.0$) would become tangent if expanded to other lift coefficients then determines the point where stalling is predicted to start. This point is most easily found by calculating $\frac{c_{l_{max}}-c_{l_b}}{c_{l_{a1}}}$ at several points along the semispan. The minimum value gives the location of the predicted stalling point of the sections with small flap deflections. The pitching-moment coefficients are in good agreement except for the N. A. C. A. 5-10-16 wing with 0.5b flap.

The C_{D_e} curves given in figures 13 to 17 are in best agreement in the region of $C_{D_{e_{min}}}$. The divergence for higher and lower lift coefficients is more for these wings than for wings without flaps (reference 4).

Two values of C_{D_e} are listed in table I, $C_{D_{e_{min}}}$ and C_{D_e} at $C_L=0.7$. It is interesting to note that, for the N. A. C. A. 23012 wing, C_{D_e} increases with flap length up to $b_f/b=0.5$ but is then substantially the same at $b_f/b=0.7$ as it is at 0.5. The reason for this variation is that the increase in profile drag with flap length is compensated by the reduction in induced drag beyond $b_f/b=0.5$. If plain flaps at a moderate angle are used

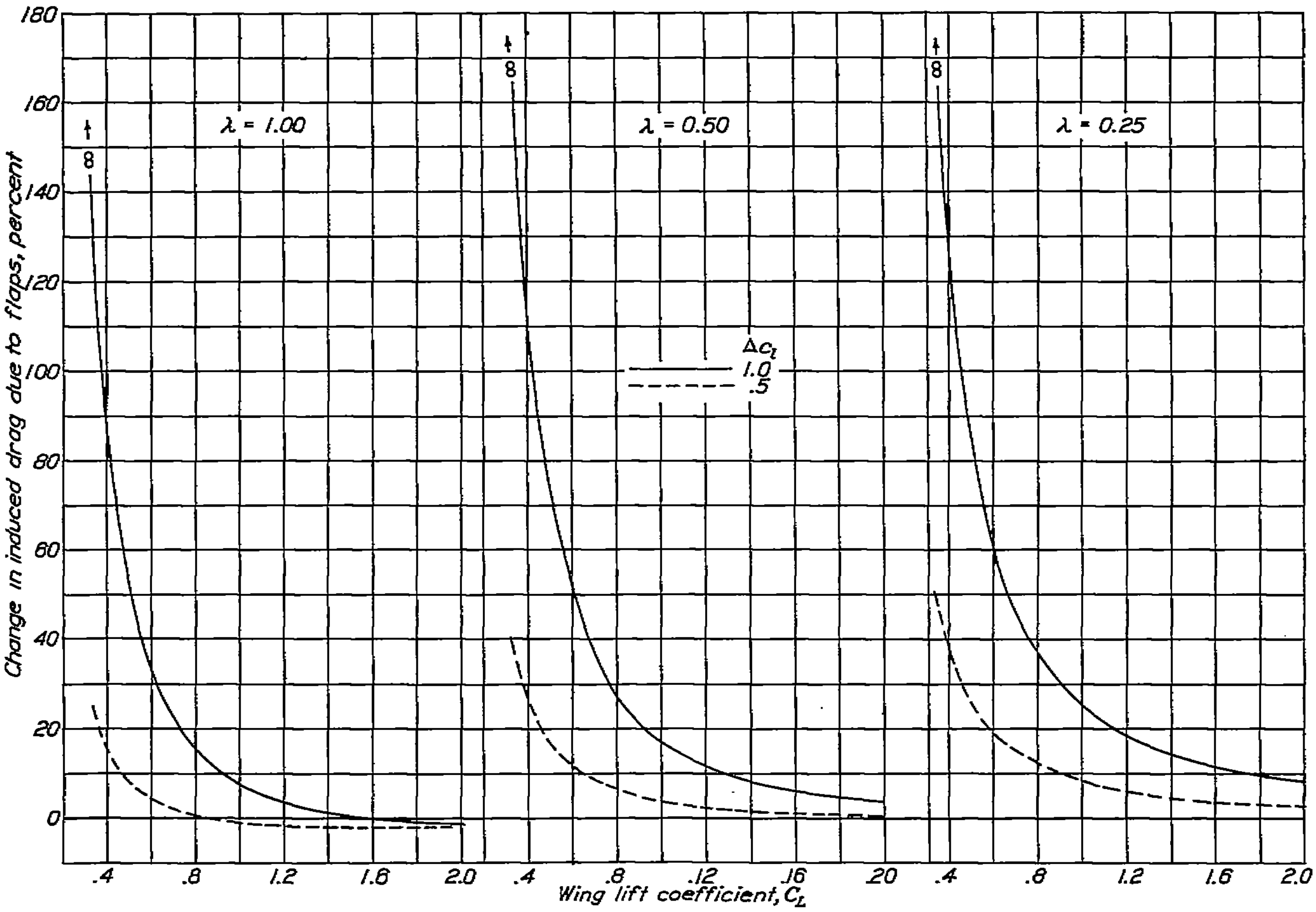

FIGURE 27.—Percentage change in induced drag relative to that for flaps neutral. 0.5b plain flap; A, 6
Percentage change given by $\frac{C_L \Delta c_l v + \Delta c_l^2 w}{C_L^2/\pi A u} \times 100$.

for take-off, as long a flap span as possible is therefore indicated to obtain the lowest drag. The decrease in induced drag with flap length beyond b_f/b equal to about 0.36 is illustrated in figure 26. The curves are given for various taper ratios, two values of A, and for C_L and Δc_l both equal to 1.0. A value of $\Delta c_l = 1.0$ corresponds to an airfoil of moderate thickness with a 0.2c flap at an angle a little larger than 20°. The change of induced drag with lift coefficient is illustrated in figure 27 for a wing with half-span flap of aspect ratio 6 and for two values of Δc_l.

The calculated and the experimental $C_{L_{opt}}$ values given in table I are in as good agreement as can be expected in view of the difficulty of determining $c_{l_{opt}}$. The agreement of the $C_{L_{max}}$ values is fair.

CONCLUDING REMARKS

Although test results and the comparison with calculated results have been given only for the case of a plain flap deflected 20°, other test results were available for the N. A. C. A. 23012 wing with a split flap 45° down and with the same flap spans as those used herein. Comparison of the test and the calculated results for the split flap showed agreement similar to that obtained for the plain flap deflected 20°. It thus appears that fair estimates of the characteristics of tapered wings with partial-span flaps deflected various amounts can be obtained from the factors and the method given.

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LANGLEY FIELD, VA., *January 23, 1939.*

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APPENDIX A

SYMBOLS USED IN TEXT

c_l, section lift coefficient.

Δc_l, increment of section lift coefficient due to flap deflection.

c_{l_b}, section basic lift coefficient ($C_L=0$).

c_{l_a}, section additional lift coefficient.

$c_{l_{a1}}$, section additional lift coefficient for $C_L=1.0$.

c_{d_0}, section profile-drag coefficient.

$\alpha_{s_{(L=0)}}$, wing angle of attack for zero lift, measured from the root chord.

$\alpha_{l_{0_s}}$, angle of zero lift of root section.

b, wing span.

b_f, total flap span.

S, area of wing.

S_f, area of part of wing equipped with flaps.

A, wing aspect ratio, b^2/S.

λ, taper ratio, c_t/c_s.

q, dynamic pressure.

c, chord at any section along the span.

c_t, tip chord (for rounded tips, c_t is the fictitious chord obtained by extending the leading and the trailing edges to the extreme tip).

c_s, chord at root of wing or plane of symmetry.

Λ, angle of sweepback measured between the lateral axis and a line through the aerodynamic centers (approximately the quarter-chord points) of the wing sections.

δ_f, flap angle.

C_L, wing lift coefficient.

C_D, wing drag coefficient.

C_{D_0}, wing profile-drag coefficient.

C_{D_e} effective wing profile-drag coefficient.

C_{D_i}, wing induced-drag coefficient.

$c_{m_{a.c.}}$, section pitching-moment coefficient about section aerodynamic center.

$c_{m_{(a.c.)_0}}$, section pitching-moment coefficient about aerodynamic-center position with flap neutral.

c_{m_0}, section pitching-moment coefficient with flaps neutral.

Δc_m, increase in section pitching-moment coefficient above c_{m_0} due to flap deflection.

M, total wing pitching moment.

M_{l_b}, wing pitching moment due to basic-lift forces.

$C_{m_{l_b}}$, wing pitching-moment coefficient due to basic-lift forces.

C_{m_s}, wing pitching-moment coefficient due to the pitching moments of the wing sections.

$C_{m_{a.c.}}$, total wing pitching-moment coefficient about aerodynamic center.

a, wing lift-curve slope.

a_0, lift-curve slope of section without flap.

a_{0_f}, lift-curve slope of section with flap.

x, moment arm measured from the quarter-chord point of the root chord and parallel to it (positive rearward).

y, lateral distance.

y_f, lateral distance to inboard end of flap.

$x_{a.c.}$, coordinate of wing aerodynamic center.

R_e, effective Reynolds Number.

L_a, additional load parameter.

L_b, basic load parameter.

J, factor of angle of zero lift.

H, factor of wing aerodynamic center.

G, factor of basic-lift pitching moment.

f, factor of wing lift-curve slope.

E and E', factors of section pitching moment.

u, v, w, factors of induced drag.

APPENDIX B

AERODYNAMIC FACTORS IN TERMS OF THE FOURIER COEFFICIENTS

The various aerodynamic factors were obtained from a Fourier analysis in which the circulation Γ was expressed (see reference 2) by

$$\Gamma=\frac{c_s m_s V}{2}\Sigma A_n \sin n\theta$$

where

c_s is the chord at plane of symmetry.

m_s, slope of the section lift curve at the plane of symmetry, per radian.

V, wind velocity.

$\cos\theta=\frac{-y}{b/2}$

If the Fourier coefficients of the plain wing at an angle of attack of one radian are denoted by A_n and if the Fourier coefficients for the same wings with a constant angle of attack extending over only the center of the span are denoted by a_n, the various aerodynamic factors (in terms of the Fourier coefficients) can be found from the following equations:

$$J=\frac{a_1}{A_1 m_0}$$

in which m_0 is the slope of the lift curve at any section, per radian.

$$H=\frac{2}{\pi A_1}\left(\frac{A_1}{3}+\frac{A_3}{5}-\frac{A_5}{21}+\frac{A_7}{45}-\frac{A_9}{77}\cdot\cdot\cdot\right)$$

$$G=\frac{c_s A}{2b}\left[\left(\frac{a_3}{5}-\frac{a_5}{21}+\frac{a_7}{45}+\cdot\cdot\cdot\right)-\frac{a_1}{A_1}\left(\frac{A_3}{5}-\frac{A_5}{21}+\frac{A_7}{45}+\cdot\cdot\cdot\right)\right]$$

$$u=\frac{1}{1+\frac{1}{A_1^2}\sum_{3,5,7}^{\infty}nA_n^2}$$

$$v=\frac{c_s}{2bA_1}\sum_{3,5,7}^{\infty}nA_n\left(a_n-\frac{a_1}{A_1}A_n\right)$$

$$w=\frac{\pi A c_s^2}{16b^2}\sum_{3,5,7}^{\infty}n\left(a_n-\frac{a_1}{A_1}A_n\right)^2$$

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TABLE I

COMPARISON OF CALCULATED AND EXPERIMENTAL CHARACTERISTICS

[Wings with 0.2c plain flaps 20° down]

Wing	Flap length (fraction span)	$\lambda = c_t/c_s$	Aspect ratio	Root section, N. A. C. A.	Construction tip section (extreme tip), N. A. C. A.	Sweepback (deg.)	$\alpha_{s(L=0)}$ (deg.) Experimental	$\alpha_{s(L=0)}$ (deg.) Calculated	a Experimental	a Calculated	$\frac{x_{a.c.}}{S/b}$ (1) Experimental	$\frac{x_{a.c.}}{S/b}$ (1) Calculated
Tapered N. A. C. A. 23012	0	0.5	6	23012	23012	9.67	−1.3	−1.2	0.075	0.076	0.210	0.219
	.3						−4.8	−4.6	.072	.073	.209	.219
	.5						−6.9	−6.7	.070	.071	.201	.219
	.7						−8.7	−8.6	.067	.069	.193	.219
Tapered N. A. C. A. 5-10-16	0	.2	10	23016	23009	0	−1.2	−1.1	.088	.088	−.011	0
	.5						−6.2	−6.4	.079	.078	−.021	0

Wing	$C_{m_{a.c.}}$ Experimental	$C_{m_{a.c.}}$ Calculated	$C_{D_{0_{min}}}$ Experimental	$C_{D_{0_{min}}}$ Calculated	$C_{L_{opt}}$ Experimental	$C_{L_{opt}}$ Calculated	$C_{L_{max}}$ Experimental	$C_{L_{max}}$ Calculated	C_{D_0} at C_L=0.7 Experimental	C_{D_0} at C_L=0.7 Calculated
Tapered N. A. C. A. 23012	−0.014	−0.008	0.0076	0.0071	0.13	0.09	1.71	1.67	0.0106	0.0100
	−.068	−.068	.0165	.0155	.20	−.02	1.83	1.77	.0182	.0209
	−.106	−.105	.0175	.0170	.20	.03	1.91	1.81	.0205	.0276
	−.143	−.142	.0156	.0158	.16	.05	1.98	1.90	.0203	.0222
Tapered N. A. C. A. 5-10-16	−.009	−.007	.0080	.0081	.08	.10	1.50	1.49	.0123	.0118
	−.136	−.157	.0190	.0178	−.12	−.28	1.77	1.74	.0262	.0274

[1] Measured from the quarter-chord point of the root chord, positive toward the trailing edge.

TABLE II

CALCULATION OF C_{D_0} FOR C_L=0.8

[N. A. C. A. 23012 tapered wing with 0.5b flap deflected downward 20°; R_e (based on S/b)=8,200,000]

1	2	3	4	5	6	7	8	9	10	11	12
Distance from center, fraction semispan, $\frac{y}{b/2}$	$\frac{\text{thickness}}{\text{chord}}$ t/c	Chord c (in.)	Effective Reynolds Number R_e (millions)	$(c_{l_{max}})_{std}$ at R_e=8,200,000	$\Delta c_{l_{max}}$ (fig. 22)	$c_{l_{max}}$ (5)+(6)	$(c_{d_{0_{min}}})_{std}$ at R_e=8,200,000	$c_{d_{0_{min}}}$	$c_{l_{opt}}$	$c_{l_{max}}-c_{l_{opt}}$	Load parameter L_a (reference 2)
0.00	0.12	6.82	11.18	2.06	0.02	2.08	0.0128	0.0125	0.10	1.98	1.291
.15	.12	6.31	10.35	2.06	.02	2.08	.0128	.0126	.10	1.98	1.263
.30	.12	5.80	9.51	2.06	.01	2.07	.0128	.0127	.10	1.97	1.191
.45	.12	5.28	8.66	2.06	0	2.06	.0128	.0128	.10	1.96	1.107
.60	.12	4.77	7.82	1.75	−.01	1.74	.0071	.0072	.10	1.64	.995
.70	.12	4.43	7.26	1.75	−.02	1.73	.0071	.0072	.10	1.63	.908
.80	.12	4.09	6.71	1.75	−.03	1.72	.0071	.0072	.10	1.62	.789
.90	.12	3.50	5.74	1.75	−.06	1.69	.0071	.0074	.10	1.59	.607
.95	.12	2.66	4.36	1.75	−.10	1.65	.0071	.0076	.10	1.55	.447

1	13	14	15	16	17	18	19	20	21	22	23
$\frac{y}{b/2}$	$\frac{S}{cb}L_a=c_{l_{a1}}$	Load parameter L_b (reference 2)	$\frac{\Delta c_l S}{cb}L_b=c_{l_b}$	$C_L \times c_{l_{a1}}=c_{l_a}$	c_l	$c_l-c_{l_{opt}}$	$\frac{\lvert c_l-c_{l_{opt}}\rvert}{c_{l_{max}}-c_{l_{opt}}}$	Δc_{d_0}	c_d (9)+(20)	$c_{d_0}c$	$c_{l_{max}}-c_{l_b}$
0.00	0.946	0.289	0.163	0.757	0.920	0.820	0.414	0.0117	0.0242	0.1650	1.92
.15	1.001	.276	.168	.801	.969	.869	.439	.0127	.0253	.1596	1.91
.30	1.027	.235	.156	.822	.978	.878	.446	.0129	.0256	.1485	1.91
.45	1.048	.030	.022	.838	.860	.760	.388	.0106	.0234	.1236	2.04
.90	1.043	−.190	−.153	.834	.681	.581	.354	.0023	.0095	.0453	1.89
.70	1.025	−.252	−.219	.820	.601	.501	.307	.0018	.0090	.0399	1.95
.80	.964	−.266	−.250	.771	.521	.421	.260	.0013	.0085	.0348	1.97
.90	.867	−.224	−.246	.694	.448	.348	.219	.0010	.0084	.0294	1.94
.95	.840	−.168	−.243	.672	.429	.329	.212	.0009	.0085	.0226	1.89